# Problem Set 4

Due on Thursday, December 1st, 2016

In the following, as in class, we use $F$ to denote the cumulative density function of a probability distrubtion, and $f$ its derivative.

1. A *simultaneous first price item auction* is a way to run a combinatorial auction. Each bidder simultaneously places $m$ bids on the $m$ items, and then each item goes to the bidder that placed the highest bid on it. Each bidder then pays her bids on all the items she won.

   Show that any pure Nash equilibrium in a simultaneous first price auction constitutes a Walrasian equilibrium. (In this Walrasian equilibrium, the price of each item is the price paid by the winner for that item in the equilibrium of the simultaneous item auction, and the allocation is the same as in this equilibrium.)

2. Show that, when bidders' values are drawn independently from regular distributions, the second price auction with lazy monopoly reserves extracts weakly less revenue than the second price auction with eager monopoly reserve prices.

   (Recall that in a second price auction with lazy reserve prices, the highest bidder is offered a price equal to the maximum between her reserve price and the second highest bid. In a second price auction with eager reserve prices, all bidders below their reserves are removed first, and then among the remaining bidders, the highest one is charged a price equal to the maximum between her reserve price and the second highest remaining bid. The monopoly reserve price for a bidder is the posted price that extracts the most revenue from this bidder.)

   **Hint:** Consider the revenue from each bidder when conditioning on the other bidders' values.