# Problem Set 3

Due on Thursday, November 17, 2016

In the following, as in class, we use $F$ to denote the cumulative density function of a probability distrubtion, and $f$ its derivative.

1. For a probability distribution, the term $\frac{f}{1-F}$ has the name *hazard rate* (from the context of survival analysis). A valuation distribution is said to have Monotone Hazard Rate (MHR) if $\frac{f(v)}{1-F(v)}$ is nondecreasing in $v$.

   (a) Show that an MHR distribution is regular. (You will get no point if your answer contains more than three sentences.)

   (b) Derive the form of a family of distributions (on $[0, \infty)$) whose hazard rate is a constant.

   (c) Show that, for a buyer whose value is drawn from an MHR distribution, at the revenue optimal posted price, the buyer buys with probability at least $\frac{1}{e}$.

   (Hint: Observe that $\frac{f}{1-F} = [-\log(1-F)]'$. Also, how do we find the optimal posted price, in terms of virtual value?)

2. Myerson's mechanism easily generalizes to settings where the outcome space is not the allocation of a single item. In this example, we consider the "public project" problem. Suppose there are $n$ bidders, and there is one public project to be built. Each bidder $i$ has a value of $v_i$ for the project. The difference from the single item auction is that, if the project is built, all bidders benefits from it, otherwise no bidder gets any value. Let $v_1, \ldots, v_n$ be i.i.d. drawn from the uniform distribution on $[0, 1]$.

   (a) Describe the revenue-optimal mechanism.

   (b) Give an asymptotic (in terms of $n$) analysis of the expected revenue of the revenue-optimal mechanism.